SEIGNIORAGE, BARRED CULLING AND WISDOM OF «AL MARCO» COINAGE

«Where would a wise man hide a leaf? In the forest...
If there were no forest, he would make a forest. »
G.K. Chesterton. The sign of the broken sword.

FOR INTRODUCTION

If you are sure that medieval people had difficulties with mathematics and do not want to know that merchants’ manuals dealing with change of coins operated proportions that surpassed precision of currency trading at FOREX¹, you may not read this article. If you are sure that concern of medieval financiers was not prosperity of the state but easy money that seemed to be done with spoiling currency you may put it aside. If the historian is not sure in these he can try to understand beneath short mathematic exercises that have already been solved by the medieval statesmen and artisans.

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In past decades metrological analysis has become a numismatic tool enjoying full rights in science. Weights of coins happen to be a basis for establishing typology and chronology of issues; comparison of weights of coins issued by different sovereigns allows starting reconstruction of financial systems, which existed in states, vanished without significant financial documentation to leave. Outlines of monetary reforms, mentioned in different documents and chronicles can also be reconstructed with help of the weight analysis. The general line of the investigation in every such a particular case is medieval economy, medieval finances and medieval currency as the part of finances and economy.

There is no use to question here, why in the Middle Ages gold and silver were used to measure values. They were indeed the world equivalents, and this question is a question to the whole mankind and to history of civilization. Nevertheless this peculiarity of the human mind was one of the main components of the medieval coins. Of course, there were Chinese, who as early as XIII c. have already used paper money, or Byzantium and Muslims states, where divinity and tyranny of the power made it possible to keep the Roman tradition of copper and brass coinage. Of course these coins were just change and the main circulating media at the East was based on gold and silver as well. Coins had to contain gold or silver to serve a measure of value, and content of precious metal was the main factor that determined the value of the coin. But coins had also the added value. Coinage took expenses to pay workers and officials, to buy coal or tools, and these expenditures were the reasons why the price of raw silver or gold was to be below the price of minted silver and gold. However a piece of gold designed in any manner with dice would have stayed a piece of metal

¹ In the example «A 58 ¼ gigliati per fiorini 5 d’oro viene l’uncia sol. 10, den. 3 e 141/233» precision of the rate is 0.00211%. — Pegolotti F. B. La pratica della mercatura / ed. by A. Evans. Cambridge (Mass), 1936. P. 172; A hundred years later merchants recommended to calculate rates of coins up to one grain (1/6912 of a pound, i.e. 0.0145%) — Saminiato de’Ricci. Il manuale di mercatura. Genova, 1963. P. 136. Practicing merchant like Badoer took into account at least difference between rate 11 and 4/17 Ottoman aqches for Byzantine perper and 11 and 4/18 aqches for perper, i.e. 0.117%. — Il libro dei conti di Giacomo Badoer, Costantinopoli (1436–1440) / Ed. U. Dorini, T. Bertelè. Roma, 1956. Fol. 18v, «aver» 01, entry 02.
and would have never become a coin if there had not existed a «public convention». It implied that the quality of the minted media was guaranteed. Thus seigniorage (that were the core of this convention or usurpation, if you like more) happened to be not only a privilege, which made it possible for a state to obtain profit from coinage, but also a duty to guarantee quality and the standard of the circulating media. Overvaluation of coins at the mint was not obligatory but it was quite natural and rather common way to increase state incomes. Seigniorage made minted gold and silver even more costly.

The amount of overvaluation was in fact the matter of fiscal policy and the financial system adopted in the state, but it was also the matter of technology. If in the Middle Ages could exist monetary systems where coins were current by weight and overvaluation of minted metal guaranteed only alloy of coins? Merchants checked the weight of acquired coins in wholesales going abroad, but the dominant approach was money counting. The best proof of this practice is clipping and culling of coins, which would have been absurd if coins had been current by weight.

It was possible to reckon specie without weighing just in case if identical coins had identical values. Which value, for unlike copper or brass coins those containing gold and silver had two values? It could be only the face value of the coin, because the intrinsic value has depended on the coin’s weight and alloy and thus has inevitably varied. These variations, as it should be evident already for numismatists, were submitted to the Gaussian law, which describes variations of parameters for infinite number of uniformly manufactured articles.

The difference between the intrinsic value of coins and the face value of their issue was a trigger ready to pull clipping and culling. The folk always yened to get easy money — to cut edges of coins, to pick out heaviest ones, to re-melt them and sell this gratuitous silver. To avoid these abuses the nominal value had to surpass intrinsic value of coins; in principle (to secure absolutely stable currency) it had to surpass intrinsic value of any coin in the issue. The consequence of this reasoning is a little bit unexpected — seigniorage and overvaluation of coins had appeared not because the wish or cupidity of sovereigns. They were fiscal measures absolutely necessary to maintain stability of the currency, to prevent reduction of weight of circulating specie in average. By this reason remedy and overvaluation of coins depends on each other, and without calculations one can assume a face value of the issue — the weight of the heaviest coin.

Culling was a crime and it is hard to find traces of culling in numismatic material. They can come into view if a monetary system had survived serious changes, some reforms. The Byzantine currency of the beginning of the XIII c. gives us a rare example; it is the hoard containing 142 silver trachea of Nicaean emperor Theodor I Lascaris (1204–1222), found in 1977 somewhere in Turkey. Simon Bendall and David Sealwood used this hoard to determine and demonstrate that Byzantine scyphate coins were produced with use of three dice. They undertook also die study to reconstruct sequence of dice usage which allows explain here some peculiarities of Nicaean financial affairs.

Figure 1 drawn with data obtained from the publication, demonstrates that Nicaean coins were minted at least by two standard weights. Varieties produced with dice «Ae–Ib2» and evidently variety «Ja» (which has to precede «Ic1») had the legal weight over 3.9 g, and coins struck with dice «Ic1–Ih» — under 3.9 g. About 20 coins from the first group of dice are under the said weight, but it is likely due to clipping.

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2 Perhaps the most famous example is Arab dirhams of the IX–XI c. that were current by weight in the Eastern Europe because there were no local coinage, no powers to conclude this «public convention», at last, no need for it.
Fig. 1. Weights of Theodor I Lascaris silver trachea

Trachy (silver, 4.36 g). Byzantium, Nicaea, Theodor I Lascaris (1208–1222). SB 2064; DOC IV.2.3–4. (LHS Numismatik; Auction 97, lot 105). Pre-reform issue, die variety «Hb2».

The weight distribution of these coins cannot be completely explained with the analysis of the histogram, although it is evident even from the histogram as well as from the figure 1, that there were at least two legal weights under Theodor I. To explain the situation we need to resort to Gaussian distributions, to be exact, to integral Gaussian distributions (figure 2). The approximation of weights of silver trachea with the sum of two Gaussian distributions became possible only if we accepted that one the issues had lost a part of its heaviest coins.

![Fig. 2. Cumulative distribution of weights (Nicaea, Theodor I Lascaris, silver trachea); Correlation index 0.9995133.](image)

The best results are achieved if we suppose that (about) 10% of trachea above 4.36 g had disappeared from circulation and have not been hoarded. These coins were those that hadn’t been clipped but had been culled after the monetary standard had been changed. The result of the approximation (it ignores 7 super light coins) is exhibited at the figure 3. The first weight standard

![Fig. 3. Gaussian laws describing distribution of weights (Nicaea, Theodor I Lascaris, silver trachea).](image)

Analysis of the histogram (it is attached to fig. 3) cannot take into account mutual influence of two weights and absence of heaviest coins.
was about 4.18 g; the second was a little bit over 3.6 g. And we can see also from these figures that there was a moment in Nicaean financial history, when it was profitable and safe to pick up coins, but only coins over 4.36 g had suffered from this culling.

Disappearance of trachea that surpass this level only means that people could not gain profit from coins with weight less than 4.36 g. It was impossible because Nicaean coins had added value, so relation between level of culling and the second legal weight 3.6 g tells us that Theodor I Lascaris introduced new trachea with added value about 21.66%. Another statement will also be correct: face value of the Nicaean trachea was equal to 4.36 g of raw silver of the same alloy. The latter is a chink that allows us to peep into the monetary system of Byzantium after the Fourth Crusade: Theodor I issued a coin with face value equal to two Venetian silver ducats that had legal weight 2.18 g. If you can convince me it was an accident that Byzantines adjust their trachea to the coin that became in the XIII-XIV centuries the standard in Levantine trade?

In general the accepted level of overvaluation and amount of seigniorage were a matter of fortune and fiscal management. Powers controlling mines had a wide choice; others were in competition attracting import of silver and gold to their mints. Statesmen faced a common management dilemma: they could order a low price of coinage to stimulate rise of the mint production and thus increase incomes of the state. Or on the contrary they could have introduced a high price of coinage in hope that it would not scare away from the mint most of importers. Their choice had impact on mint technology: slight overvaluation demanded precisely weighted coins, high amount of seigniorage allowed negligent coinage with huge remedy.

Technological limitations to the ideas of financiers have appeared only in case if the mint had to produce very small coins. In reality these limitations were not technological but economical — process of weight justification became expensive if it was applied to coins of petty value. Although medieval tools were not as perfect as modern ones, workers who cut blanks could produce with primitive scales and scissors coins that had weight adjusted much better than in plenty of modern coins. An example of available precision one can find in the famous medieval coins — gold ducats of Venice. They were famous for their stability that was maintained for ages. Venetian mint regulations have foreseen that the weight of 67 ducats was over one mark and under 1 mark 2 karats (238.5–238.92 g). Thus the variation of weight had to be \( (238.5–238.92):(238.5+238.92)=±0.088\% \). As this was the variation for joint weight of 67 coins, by the law of statistic the variation of weight for single coin was \( \sqrt{67} \) times more, i.e. \( ±0.72\% \). In absolute figures a gold ducat could vary by weight just by \( ±0.0257 \) g from the legal weight \( (238.5+238.92):2:67=3.5628 \) g.

It is hard to correlate Venetian regulations with a mathematical law, as we don’t know how much coins surpassed prescribed limits. If we suppose that 99.73% of coins were produced by the said remedy we admit that allowed remedy corresponds to 3 standard deviations \( \sigma \) of Gaussian law (the common limit in statistics). Then standard deviation of gold coinage in Venice should be calculated as \( \sigma=0.00855 \) g. The gold ducat is, of course, a test case, no specimens in theory surpassed 1% of the legal weight, and any culling was impossible with the price of coinage about 1%. They could only clip these coins, and they did, but clipping is not the subject of this article.

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7 Stahl A. M. Zecca… P. 233. — Leonardo Gradenigo lost his right hand and both eyes in 1413 for clipping ducats in spite 20 years have passed since committing the crime (3 Mar. 1393: ASV. AC Raspe, R. 3645/5, fasc. 1, f. 1).
IN VENICE

In the last quarter of the XIVth century Venetian authorities were anxious about culling of soldini in the city. Most likely the problem has appeared after the balance between overvaluation and remedy of coins had been disturbed, when the price of coinage was reduced in 1379 from 8% to 2.6%. To improve the situation with the currency the Senate tried to impose new mint regulations that required manufacturing of blanks between 63 and 62 coins per ounce. However emenders who worked on soldini at the mint refused to produce coins that had to have such a low remedy of ±0.038 g (±0.8%). After series of inspections a compromise has been achieved and the weight range of individual soldini was set from 61 to 66 coins per once (0.451–0.489 g, remedy ±0.0185 g or ±3.88%), and aggregate average held at 63.25 to 63.75 coins per ounce. The latter assumes different but more realistic figures of remedy (calculated in the same manner as for ducat) — ±0.0451 g or ±9.45%. In any case, it was sufficient to prevent culling in next decades, and there were no complains on it in the documents.

The problem, which the mint had to solve with soldini appeared as a result of financial policy adopted in the state. Soldini were produced from so called «quinto» silver (obligatory sale to the mint 20% of all imported silver by the understated price) and their overvaluation was 22.4%, that seems to contradict their trifling remedy. However Venetian mint strategy in the end of the XIVth century was quite extraordinary phenomenon of the Middle Ages: the Republic struck another coin — grosso, from «free» silver without seigniorage, and even more, mint overheads and some expenses were compensated with incomes from coinage of soldini. This was the way the Venetian Senate had solved many problems — foreign merchants were persuaded to change their silver bullion into grossi and citizens — to turn their silver wares into coins. There was an urgent need in specie as it has been floating from the state with abnormal velocity. Thus remedy of soldini was tuned just to this policy of almost gratuitous coinage of grossi and we have to take into consideration the latter.

* * *

The hoard of Venetian grossi and Mamluk dirhams has attracted my attention, when investigating absolutely different problem — the outflow of silver. The hoard contained 1077 dirhams dated 655–807 AH (1255–1405), the bulk of Venetian coins were 43 grossi of the 2nd and 640 of the 3rd type (June 1394–November 1401) issued by Antonio Venier (1382–1401). According to the hoard’s data Venetian currency was extremely mobile in this period, about 33.3% of coins left...

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9 Stahl A. M. Zecca… P. 198.
11 Stahl A. M. Zecca… P. 186.
the circulation every year\textsuperscript{14}, so the average lifetime of coins in circulation could be only 3 years and consequently in general they were practically out of wear.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{grosso.png}
\caption{Grosso, 3rd type, (silver, 1.78 g). Venezia, Antonio Venier (1382–1401) Papadopoli 229, 3; Paolucci 37, 3; Biaggi 2855. (Jean Elsen & fils. List 233, no. 910).}
\end{figure}

Weight analysis of these grossi demonstrates the difference between two varieties of these coins. Grossi with the «leonine» head of St. Mark and grossi with the «conventional» head of the saint correspond to the change of the weight standard from 127 coins per mark (1.878 g) to 131 coins per mark (1.821 g) that happened, according to A. Stahl, in 1397\textsuperscript{15}. Modal weights $M_0$ of these varieties in the hoard make up 1.836\textsuperscript{16} and 1.786\textsuperscript{17} g and standard deviations $\sigma$ of the issues are equal to 0.03448 and 0.03722 g (remedy ±5.51% and ±6.13%).

\textsuperscript{14} To determine by a hoard an approximate value of $\lambda$ — «probability of extraction» that could exist during someone’s reign we have to take logarithmic value of the proportion and to divide it by the duration of the reign. The proportion itself is not sophisticated:

\[
P = \frac{\text{coins of preceding rulers}}{\text{all coins but coins of posterior rulers}}
\]

This simplified method for calculation derived from the analysis of about 300 hoards and different currencies, that confirmed that outflow of coins was an average constant for a particular currency. «Probability of extraction» is the parameter of exponential law $N_t = N_0 e^{-\lambda t}$ that determines how much coins have left by some moment $t$ in the circulation from the initial amount $N$. In case of the analysed hoard it makes up:

\[
\lambda = \frac{\ln(691)}{51} = 0.4062
\]

Average lifetime of coins in circulation is an inverse parameter to «probability of extraction», i.e. $1/\lambda$.

\textsuperscript{15} The weight of grosso was officially reduced after 7 October 1399, although this was a mere recognition of a weight reduction, which had in fact taken place two years previously. — Stahl A. M. Zecca... P. 77; Bonfiglio Dosio G. Capitolar dalle Broche... P. 66; M. Phillips M., S. Tyler-Smith. A Hoard of Venetian Grossi ... P. 273. The authors of the article stressed that weights of coins from the hoard contradict the idea of A. Stahl that the change had happened with the change of punches for the doge portrait from «beardless» to «bearded» type (Stahl A. M. A Fourteenth Century Venetian Coin Portrait. ANSMN 30, 1985. P. 214). However, slight disturbances in distributions make us say that the correspondence was not strictly typological. During a short transition period «leonine» and «conventional» varieties were used for coinage by the foot 127 coins per mark and by the reduced foot of 131 coins per mark, because the light variety has small surplus of heavy specimens, and the heavy one, on contrary, surplus of light specimens.

\textsuperscript{16} 139 coins from 147 listed; coins with weight under 1.72 g excluded together with mounted, ex-mounted and with signs of mounting. Correlation index is equal to 0.99347.

\textsuperscript{17} 446 coins from 469 listed; coins with weight under 1.69 g excluded together with mounted, ex-mounted and with signs of mounting. Correlation index is equal to 0.99546.
These figures are parameters of the Gaussian laws that describe coinage of grossi in the days of Antonio Venier. Since we know them we may see how much silver would have contained picked up coins if culling had taken place (standard of silver fineness specified in November 1400 was 952‰). The figure 3 demonstrates the alteration depending of the minimal weight of culled coins of «conventional» variety. The curve is, of course, theoretical one, as we understand that there were very few coins beyond remedy of $3\sigma$ (1.9323 g). In theory one coin from a thousand and a half surpassed this limit and there is just one in the said hoard.

![Fig. 4. Average silver contents of culled grossi (Venice, Antonio Venier, 3rd type, «conventional» variety)](image)

Besides, the curve is not very informative, because it gives no impression about the number of culled coins. It does the figure 4, which depicts how the average silver content of culled coins depends on their number.

![Fig. 5. Relative profit from culling as average silver content of culled coins (Venice, Antonio Venier, 3rd type grossi, «conventional» variety)](image)

18 Stahl A. M. Zecca… P. 357; Bonfiglio Dosio G. Capitolar dalle Broche… P. 66–68.
The curve is the result of modeling the process of culling with use of the mentioned parameters of the issue; it tells us, for example, that heaviest 50% of «conventional» coins contained silver 1.6% more than there were silver in the standard grosso. However culled coins had to be remelted and those losses during refining were losses of a man and not of the mint. Contemporary merchants estimated that about 0.67% of silver was lost during remelting of Venetian grossi (18 d. per libra of silver valued 237 s. 9 d.\textsuperscript{19}). However again, a man, who decided to spoil currency, expected profit. He could, for example, pick up 5% of heaviest coins that contained 4.22% silver more than standard coins and his maximal profit would make up 4.22% from these 5%, i.e. 0.211% from the total of coins inspected. It could be less than two grossi from every thousand because of refinement losses.

Thus abuse was possible, but culling of grossi seemed to make sense just if you could check many thousands of them, if you could earn a couple of gold ducats. Culling in Venice in 1390\textsuperscript{th} was not impossible but was not easy. It could not be the habit of the ordinary folk, who earned a thousand grossi a year; just people famous in the city for their prosperity had resources to practice it.

But besides there were merchants participating in international trade, primary sense and mentality that whispers that it was advantageously to export culled coins and no matter how much of them. How to deal with it? For merchants who exported silver the profitable level of culling was set not by the price of coinage in Venice, but by the price of coinage there where they went and where they intended to sell Venetian specie as bullion. Venetian mint in 1390\textsuperscript{th} had to do nothing to prevent this abuse. Grossi were protected from culling aimed to export because they were already protected in the currency with minimal possible overvaluation.

I had paid so much attention to Venetian grossi because archives and publications supply us with excellent metrological material, figures of brassage and wages that are necessary for complete explanation. The total charges to merchants for coining their «free» silver into grossi made up £0.44 (2.2 grossi) for mark of silver (127 grossi); thus these coins had the added value of 1.7255%\textsuperscript{20}.

But the barriers for culling were set not only by the price of coinage or remelting losses. Every work has its price. Emenders at the Venetian mint did in reality the same work which those picking up the coins had to do — in professional manner they checked weights of grossi. Their wage for this work was £0.05 for mark of silver, or in other words they were paid £1 (5 grossi) for weighting 127*20=2540 coins. One more person had to participate in culling — a caster. Casters at the mint received 2 piccoli (1/24 of grosso) for every mark of silver they melted\textsuperscript{21}. Thus even if the state struck coins gratis, the probable profit from culling was reduced by the refinement losses and charges compatible to wages of emenders and casters.

The figure 6 depicts profit from culling Venetian grossi overvalued by 1.7%. With 0.67% losses during refinement and fee of 0.258% to the caster maximal profit could be received if the malefactor picked up 30.6% of coins. This profit could be obtained only from the «fresh» currency without added value, and even in this case culling would give a malefactor just about 3.6 grossi after checking 1000 coins. Necessity to pay for coinage reduced profit from culling 1000 grossi to 0.61 grosso (1.09 g of silver).

Price of culling itself, however, is not present in these calculations and on the plots. If the merchant himself spend some hours to earn one gram of silver after he had weighted 1000 coins? Could he find more profitable occupation? Or whether he hired a man to pick up heavy coins, paid him one forth of the wage of the emender (the said one gram of silver), nothing to earn for himself? No difference; they never tried; culling was not practiced with grossi of the 3\textsuperscript{rd} type, because

\textsuperscript{20} Stahl A. M. Zecca… P. 186; but cfr. ibid. p. 198, table 8.1 — only 1.6%.
Venetian authorities happened to be qualified in calculations and technology to choose a proper remedy.

Venetian soldini and grossi helped to establish that general idea about correlation between remedy of coins and their added value was correct but was a simplification. Stability and protection of the currency from culling could be secured with any amount of seigniorage. The real boundary for culling was set by labour expenses, technology and size of coins. One can take for this boundary the said 1 g of silver obtained after inspecting 1000 coins. This figure allows us analyse many other currencies to calculate the added value necessary to maintain financial stability in many other states (if we have determined the particular weight standard and remedy).

Pity to say, numismatists in general are not interested in this value; documents specifying price of coinage become scarce when we leave Europe or when we go deep into the ages. One hazardous sequence of this negligence arises in studies of medieval economic history. Scholars not knowing figures of seigniorage are familiar with two misinterpretations. They used to establish gold-silver ratio with rates of coins, although they do not know their overvaluation, thus explaining metal flows with distorted or even reversed figures. They also try to establish rate of different coins based on their weights, and again they forget that coins had the added value, in most cases undetermined.

Steadiness of the Venetian currency has been based on «jewellery» technology of coinage, specialization of craft and high skill of the personnel. The mint produced utterly uniformed coins but it was not the only way to maintain stability of the circulating media and to protect currency from abuses. This was a way predetermined by desire or necessity to produce coins from high grade silver that minimized potentialities of technological maneuver.

We know many other examples that seem to have nothing common with principles embodied in Venetian grossi as well with appropriate technology. Coins from different regions and of different epochs quite often have extremely large remedy and unstable alloy even if they pertain to the same type or were struck with the same dice. Simplified explanation of these peculiarities suggests that these were the results of financial policy, which never dreamed of a stable currency. Medieval financiers according to this explanation thought that it was the fast and normal way to spoil currency to increase state incomes. If they had to cherish hopes for the folk would be deceived, would never
be able to distinguish good and bad coins, would never discount the latter in transactions and would never cull coins?

There, of course, has to exist an explanation, which foresees that medieval statesmen were aware of state finances and were not a burglary ring. The research of Ernst Oberländer-Tarnoveanu on the Moldavian coins of the XV century, containing numerous data of their alloy, helps to find the answer. \(^{22}\)

**IN MOLDOVA**

The publication of the Roumanian scholar lists Moldavian silver coins issued by Stefan III (1457–1504) and their metrology generates many questions concerning technology and financial policy. We would be able to explain exceptional variations of weight of Moldavian groats from 0.4 to 0.85 g with wording «al marco coinage», supposing that these variations were the result of operation, which controlled weight of coins «al marco» (in this case medieval emenders checked weight of a batch of coins). Indeed, the charts accompanying the research of E. Oberländer-Tarnoveanu make clear that weight distributions of Moldavian groats and half-groats deviate from Gaussian laws.

Numismatists do not bind up «al marco» coinage with metal composition of coins. They notice that it has wide remedy, that it has no evident maximum characterizing weights of coins that can be taken for legal weight of the issue. They explain these particularities as essence of «al marco» coinage, when nobody controlled the weight of a single coin. Such an interpretation of «al marco» coinage became antiquated because it has nothing to do with the statistic law describing manufacturing processes — with the Gaussian law. «Al marco» technology by this explanation was nothing more than inaccurate coinage dealing with blanks cut in a slipshod manner. Inaccuracy, however, is unable to explain why these coins have an even distribution by weight. Whoever tries with any precision to strike identical coins will obtain a Gaussian distribution of weights because laws of probability are the laws of nature; so to expect an even distribution in coinage is akin to expect the sunrise at the West. It takes to undertake special efforts to make coins different. Making coins different means that their weights were no longer accidental and correspondingly that weight distribution no more was submitted to the Gaussian law.

Negligence in production would have resulted in a Gaussian curve with a huge standard deviation, so if we observe an even distribution of weights in some coinage we have to understand that it was a result of special efforts that had some purpose. However such an even distribution was a result of technological process, which had to result in a Gaussian distribution. Where did the contradiction appear? Most of even distributions in numismatics can be described as the sum of two or more Gaussian distributions. For example, the cumulative distribution of the type I of Moldavian groats issued by Stefan III is almost linear, almost even. Groats of the type IIa (sub-varieties «1a (a, b)» according to E. Oberländer-Târnoveanu) demonstrate similar chart. But if we suppose that they had appeared as the result of merging two Gaussian distributions we may determine parameters of these laws and quotas of coins struck by different standards. Necessary computer programs and statistical procedures that use appropriate criteria to establish best fitting between empirical data and theoretical laws are to be the instrument of numismatic study in this case. \(^{23}\)


\(^{23}\) I adjust five parameters of these distributions with the standard «Monte-Carlo» method, which generates random values and allows choosing the best combination of parameters via maximal correlation index.
Two figures demonstrate that both extremely «inaccurate» coinages (their heavy specimens surpass, as I’ve mentioned, twice by weight the easy ones) combine two sub-issues with different weight standards. Even more, although standards are different for the type I and type II groats, we can see that the proportion (between numbers of coins that follow principal and protective weight standard) is identical: 2 to 1. The principal standard makes up 80% of the protective weight for both Moldavian types; one can see also that there was the difference: coinage of the type II groats became or was more accurate24, they were produced with remedy ±0.15 and ±0.18 g in comparison to ±0.20 and ±0.25 g for the type I.

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24 The chronology of two issues was disputed in the literature; now they considered that type I groats were replaced by type II, although the starting guess of O. Iliescu was opposite. In any case, reproduction of dual weight standard in all issues rejects the idea that Moldavian authorities had intentions to forge currency.
Fig. 7. Cumulative distribution of weights (Moldova, Stefan III, type I groats, 70 specimens from no. 1–83); Correlation index 0.9981888.

Fig. 8. Cumulative distribution of weights (Moldova, Stefan III, type II groats, variety «1a (a, b)», 93 specimens from no. 116–215); Correlation index 0.9985926.

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<th>Number</th>
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<th>Protective modal weight</th>
<th>Principal modal weight</th>
<th>Protective quota</th>
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<th>Relation of modal weights</th>
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<td>Type II groats 1a (a &amp; b)</td>
<td>0.7209</td>
<td><strong>0.5751</strong></td>
<td>31.84%</td>
<td>68.15%</td>
<td>0.6215</td>
<td>68.15%</td>
<td>0.798</td>
</tr>
<tr>
<td>(216-263)</td>
<td>Type II groats (all the rest)</td>
<td>0.6723</td>
<td>0.5306</td>
<td>53.76%</td>
<td>46.24%</td>
<td>0.6067</td>
<td>46.24%</td>
<td>0.789</td>
</tr>
<tr>
<td>(264-289)</td>
<td>Type II half-groats</td>
<td>0.3519</td>
<td>0.2407</td>
<td>44.235</td>
<td>55.77%</td>
<td>0.2899</td>
<td>55.77%</td>
<td>0.684</td>
</tr>
</tbody>
</table>
Fig. 9. Gaussian laws describing distribution of weights (Moldova, Stefan III, type I groats);
1) $M_o=0.62757\, g$; $\sigma=0.08271\, g$; $\mu=0.01199\, g$; P=68.02%;
2) $M_o=0.78907\, g$; $\sigma=0.06631\, g$; $\mu=0.01402\, g$; P=31.98%.

Fig. 10. Gaussian laws describing distribution of weights (Moldova, Stefan III, type II groats, variety «1a (a,b)»);
1) $M_o=0.57511\, g$; $\sigma=0.05887\, g$; $\mu=0.00739\, g$; P=68.16%;
2) $M_o=0.72085\, g$; $\sigma=0.04903\, g$; $\mu=0.00901\, g$; P=31.84%.

«Al marco» coinage indeed had to be realized with use of two standard weights — one principal and another protective. Nevertheless coins struck by different standard weights had not to differ by their standard intrinsic value. If the principal weight was 20% less than a protective one, alloy of coins in the protective group had to be lowered also by 20%. These figures do exist in the Moldavian coinage of the type I groats. According to the quotas for the principal and protective sub-issues (2 to 1) we must expect that $2/3$ of coins have silver alloy that is 20% better than alloy of the rest $1/3$. In the issue of the type I groats two groups may be detected: alloy of 8 coins varied from $888.5\%$ to $804.5\%$, and 4 others contain from $763\%$ to $580\%$ of silver$^{25}$. The average figure in the

\begin{itemize}
\item[$^{25}$] $888.5\%$; $884\%$; $873.5\%$; $864.5\%$; $864\%$; $813.5\%$; $806\%$; $804.5\%$; $763.5\%$; $722.5\%$; $620\%$; $580\%$.
\end{itemize}

first group is 850‰, in the second — 671.5‰. Eight coins from twelve analysed match the principal sub-issue, the rest 1/3 belongs to the protective component of the coinage. It is noteworthy that the proportion between average figures of alloy is, as necessary, the same as for modal weights — 0.79 i.e. 4 to 5. It perfectly corresponds with supposed «al marco» character of the Moldavian coinage: the principal issue had the average weight 20% less than the protective one, and in the protective sub-issue quality of silver is 20% inferior to that principal.

Which were these two standards of alloy? The better one as I see was 20 karats (833‰) and the worse one — 16 karats (667‰). These figures are slightly different (1.6% and 0.4%) from the averages calculated by the results of XRF analyses, but if it is not quite a well-known phenomenon of surface silver enrichment, it is a deviation determined by the small number of analyses. According to these figures the principal and protective groats contained the same amount of silver, at least 0.7905×16:24=0.6291×20:24=0.524 g. The mixture of both sub-issues (if the foots of coinage were 300 and 375 coins from the Buda mark, equal to 245.53779 g) had to have the average weight about 0.7064 g. Average alloy (769‰) was close to 18.5 karats; coins of both standards were planned to contain 0.5434 g of pure silver and an average coin contained the same amount.

The XRF analyses cited in the publication of the Roumanian scholar allow establishing firmly the hypothesis that purposes of «al marco» coinage were protection of the currency and reduction of minting expenses by work simplification (it was most pressing with coins under the weight standards under 1 g). The evident advantages of this «untidy» coinage in comparison to European «jewellery» weighting of coins can be realized if the state was ready to issue coins with silver alloy under 937‰ (22.5 karats) and could take for coinage at least 3–4% of the total value. It can be seen at the figures identical to those exhibited for Venetian grossi. These figures demonstrate also that Stefan III groats could not survive, of course, if there had been no overvaluation of currency in Moldova. Without overvaluation profit from culling these coins was 28 groats if someone picked up 625 heavy coins from 1000.

![Fig. 11. Relative profit from culling as average silver content of culled coins (Moldova, Stefan III, type I groats).](image)

26 This figure was calculated with modal weights of the distribution, standard weights should be somewhat more. I suppose that foots of coinage for the type I grossi were 300 and 375 coins from the Buda mark equal to 245.53779 g; they correspond to 0.8185 and 0.6548 g standard weights.
Fig. 12. Profit of culling from 1000 coins (Moldova, Stefan III, type I groats); including losses in remelting and payment to the caster, without fee for culling.

But what is important, these figures demonstrate that there in Moldova has been applied financial policy that permitted to stabilise local currency in the 2nd half of the XV century without exceptional overvaluation of coins. Already 4% overvaluation made culling irrational: with the same result they could pick up 10% and 50% of groats, struck by «al marco» technology. To limit malefactors, like in Venice, with gains equal to 1 g of silver, the state had to take for coinage every twelfth coin, i.e. to set added value equal to 9.09%.

For comparision, to protect the uniformly manufactured groats with the same success as with 9.09% «al marco» overvaluation the mint had to introduce an incredible 22% price of coinage. With these 9.09% Stefan III has received for his coins an absolute guarantee from culling. This guarantee was similar to that which Venetian mint masters had secured for Republic adjusting the weight of soldini with precision ±0.0451 g, almost six times better than did men in Moldova — ±0.25 g. The Moldovian mint reduced expenditures, as implementation of «al marco» technology hasn’t change the nominal value of coins or their output, but has drastically accelerated and facilitated the most laborious operation of preparing and weighting blanks. It is hard to believe that there was a need in supplementary control and the particular profession of the emender.

Later changes at the financial market made Stefan III to introduce a new type II of the Moldovian groat. It had both weight standards reduced about 1/10 and was produced with remedy ±0.18 and ±0.15 g for principal and protective part of the issue. Increased accuracy was the demand of finance administration intended to decrease overvaluation, besides production of principal coins from high grade silver (above 23 karats) should be more accurate to prevent culling. Type II groats had received the same protection at the same level as earlier (with profit equal to 2.22 coins from 1000) if the price of coinage was reduced to 5.26% (1 coin from 20

27 A Venetian worker could cut in 1318 700 blanks for piccoli in a summer day. — Stahl A. M. Zecca... P. 340.
28 Metal composition of these groats is not as evident as for the type I, accordingly it is not quite clear, which were foots of coinage. Quite possible that principal one was the strict copy of the Hungarian standard for denars after 1467 — 416 coins from the Buda mark (0.5902 g), so mint masters could choose for the protective sub-issue weight 0.7366 g (1000 coins from 3 marks). In this case losses in ground are indistinguishable — 0.018 and 0.017 g.
29 981.5‰; 980‰; 976.5‰; 973.5‰; 969.5‰; 965‰; 963‰; 962.5‰; 962‰; 961.5‰; 960‰; 960‰; 957.5‰; 957.5‰; 951‰; 948‰; 927.5‰; 925‰; 924‰; 919‰; 918.5‰; 918.5‰; 904‰; 888.5‰; 885‰; 884‰; 864.5‰; 813.5‰; 806‰; 804.5‰; 763.5‰; 722.5‰ — E. Oberländer-Târnoveanu. Emisiunile monetare... P. 367–368.
minted). This change could be predetermined by monetary changes in one of the neighboring and mighty states — in Hungary or Turkey, in Poland or in Crimea. Like Venetians who had to tune their soldini to the coinage of grossi, Moldovan authorities should change their coins if surrounding currencies had altered.

However the face value of the Moldovan coins hasn’t changed with introduction of type II groats. Amount of seigniorage was the real difference of two types accompanied by stylistic changes of groats. Nominal value of type I groats overvalued by 9.09% was 0.5952 g of pure silver. The reduction of seigniorage for the type II groats by 4% didn’t change this figure: silver content of type II groats of 962% alloy, struck by the principal foot 416 coins from the Buda mark, was 0.5678 g and financiers calculated their face value to be immutable — about 0.5976 g of silver.

ELSEWHERE

Two standard weights with two corresponding alloys in the same issue were the main principles of «al marco» coinage. Reproduction of the proportion 2 to 1 in other currencies makes me think that this one was the most effective and most comprehensible way of doing. That was the case of the Bulgarian groats/aspers minted in Vidin by Ivan Stratimir (1360–1396) in the end of his reign. In the parcel occurred in trade in 199330 and containing 289 of his groats, 13 specimens seems to be the remnants of the previous issue31, the rest are another example of «al marco» coinage. Principal coins are twice as numerous as protective ones, protective issue is more accurate, two modal weights are in relation, close to 5 to 6. They make up 0.3906±0.108 and 0.4568±0.14 g in the hoard, so it seems to me that chosen foots of coinage were 600 and 500 from the Buda mark (0.4092 and 0.4991 g), and we can expect the corresponding difference in alloy32.

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31 Cfr.: Dimova V. Trésor monétaire du moyen age découvert près de Roussé. Bulletin de l’institute d’archeologie. Sofia. 1962. T. 25. P. 71–87. Modal weight «Mo» of this group makes up 0.6101 g, however, it can be only overestimation of the previous principal standard.
32 For example, 22.5 and 18.75 karats; in this case silver content of Ivan Stratimir groats was predetermined as 0.3837 g.
Fig. 14. Gaussian laws describing distribution of weights (Bulgaria, Vidin, Ivan Stratzimir, groats).
1) \( M_0 = 0.39059 \text{ g}; \sigma = 0.03606 \text{ g}; \mu = 0.00264 \text{ g}; P = 64.14\% \);
2) \( M_0 = 0.45681 \text{ g}; \sigma = 0.04652 \text{ g}; \mu = 0.00491 \text{ g}; P = 31.02\% \);
3) \( M_0 = 0.61011 \text{ g}; \sigma = 0.04729 \text{ g}; \mu = 0.04951 \text{ g}; P = 4.56\% \).

Groat (silver, 16.5 x 17.1 mm; 0.52 g). Bulgaria, Vidin, Ivan Stratzimir (1360–1396)
(Ancient Numismatic Enterprise; Vcoins 1689).
Trapezuntine aspers manufactured under Alexius IV Comnenus (1417–1428) that preserved in the hoard of 62 coins confirm that the demand for «al marco» protection could exist in his Empire too. The mint masters have chosen different proportions than Bulgarians and Vlachos. The protective issue was 20% of the total, but once more it was 20% heavier than the principal one. It was the most effective scheme, with overvaluation equal to 3% the malefactor could receive only one asper culling heavy coins from a thousand.

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33 Coin Hoards. 7. no. 370. The publication lists only 60 coins, 7 of them belongs to the variety with reduced to 0.74 g weight (letter «B» under the horse on obverse); weight of one more coin (0.56 g) seems to be a misprint.

34 Of course, a chemical analysis may show that it has been a simple reduction of weight of uniformed Trapezund coins. Remedy of Alexius IV aspers (±0.15 g; ±15%) made currency safe from culling with tolerable 7.5% overvaluation, which set culling profit to the said 1 asper.
Signs of implementation of «al marco» technology can be found in last Byzantine coins. We can see that two modal weights in the issue of Manuel II (1391–1425) aspers — 0.898±0.101 and 0.816±0.147 g differ exactly by 10%\textsuperscript{35}. Two groups in the familiar proportion 2 to 1 would have similar difference in alloy not yet measured (for example 22 and 20 karats, 917‰ and 833‰) and average silver content of coins from this hoard would be 0.748 g with alloy about 887‰. More than excellent combination preserved coins; with overvaluation equal to 3.6% it limited culling acquisitions to non-profitable 1.4 aspers (1 g of silver).

Most of John VIII (1425–1448) aspers in this hoard (as possible to deduct from weights of 33 coins) follow these proportions and these standards also. Principal and protective sub-issues have following parameters: a) \(\text{Mo}=0.8225\, \text{g}; \, \sigma=0.0427\, \text{g}; \, \text{P}=68.3\%\); b) \(\text{Mo}=0.9096\, \text{g}; \, \sigma=0.019\, \text{g}; \, \text{P}=31.7\%\). They are obviously identical to that of Manuel II coins. That makes to state once more that two weight standards of Manuel II aspers were not the result of debasement, but the generic property of the coinage. The debasement of the Byzantine asper has been undertaken already in the reign of John VIII and seven or eight aspers that have weight under 0.725–0.665 g can be the youngest coins in the hoard.

It should be mighty neighbors who were responsible for changes in financial policy of Bulgaria, Moldova or Byzantium. So this short list, of course, should be developed to make it possible to detect the amount of seigniorage in the surrounding states, where the secret of «al marco» coinage could also be exploited. Surprising variations of weight and alloy of coins that do not differ in typology have made scholars to suspect medieval statesmen and mint masters to be short-minded malefactors always ready to forge coins of their sovereigns. But if we would ask ourselves «Where would the wise men hide a coin?», we should answer «In the currency…» Medieval financiers were wise, as they have succeeded to hide. And we cannot consider ourselves more able just because we can explain with functions, integrals and plots the success they have achieved with these very simple proportions — 1 to 2, 4 to 5, 5 to 6 and 9 to 10.

ANSMN — American Numismatic Society. Museum Notes
ASV — Archivio di Stato di Venezia
NC — Numismatic Chronicle
SB — Sear D.R. Byzantine coins and their values
DOC — Catalogue of the Byzantine Coins in the Dumbarton Oaks Collection and in the Whittemore Collection
MBR — Buzdugan G., Luchian O, Oprescu C.C. Monede și bancnote românești, București, 1977
Papadopoli — Papadopoli N. Le Monete di Venezia descritte ed illustrate
Paolucci — Paolucci R. Le monete dei dogi di Venezia
Biaggi — Biaggi E. Monete e zecche medievali italiane : dal sec. 8. al sec. 15.
RIN — Rivista Italiana di Numismatica e Scienze Affini.